# Sheet 3

## *System of First Order and Higher Order Ordinary Differential Equations*

1. Use the **Euler method** to solve

$$\frac{dy}{dx} = -2y + 5e^{-x}$$

$$\frac{dz}{dx} = -\frac{yz^2}{2}$$

over the range x = 0 to 1 using a step size of 0.5 with y(0) = 2 and z(0)=4.

2- Solve the following 2nd order differential equation by **Euler method**.

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 5 \ , \ q(0) = 0 \text{ and } \frac{dq}{dt}(0) = 0$$

where L=1, C= 0.25, R= 0.5 for t = 0 to 0.2 with a step of 0.1

3. Solve the following 2nd order differential equation by **Euler method**. .

$$\frac{d^2x}{dt^2} + 5x\frac{dx}{dt} + t(x+7) = 0$$

$$where \ \ x(0) = 6 \text{ and } \frac{dx}{dt}(0) = 1.5$$

Find y(1) consider Δt=0.5

4. Using **midpoint method** solve the following initial-value problems for the system of first-order ordinary differential equations for $0 \le x \le 1$ and h=0.5

$$\frac{dy_1}{dx} = 2x - 3y_1, \ y_1(0) = -1$$

$$\frac{dy_2}{dx} = 3y_2 + 2x \ , \ y_2(0) = -2$$

5- Using **midpoint method** solve the following initial-value problems for the second-order ordinary differential equation

$$\frac{d^2y}{dx^2} = x\frac{dy}{dx} + y^2 \ , y(0) = 1, \frac{dy}{dx}(0) = 2,$$

use step size of 0.1 to find y(0.2).

6. Using **midpoint method** solve the following initial-value problems for the second-order ordinary differential equations for $0 \le x \le 1$, h = 0.5, y(0) = 0.5 and y'(0) = −0.5. by reducing the equations to a system of first-order ordinary differential equations

$$\frac{d^2y}{dx^2} = 3\frac{dy}{dx} + y - x - 1,$$

1 - السؤال الاول والرابع محلولين حل نموذجى
2 - السؤال الثانى والخامس سيتم شرحهم فى السكشن
3 - السؤال الثالث والسادس سيحلهم الطالب ويقدمهم فى تقرير منظم فى الموعد الذى سيحدده المعيد
4 - فى حالة تقديم التقرير بعد الموعد المحدد فلن يقبل منه مهما كانت الاعذار ولن توضع له درجة

$1 - Given$
$Let\ y_1 = y\ and\ y_2 = z, then$
$f_1(x, y_1, y_2) = \ -2y_1 + 5e^{-x}$
$f_2(x, y_1, y_2) = \ -\frac{y_1 {y_2}^2}{2}$
$x(0) = \ 0$
$y_1(0) = \ 2$
$y_2(0) = \ 4$
$x_f \ = \ 1$
$h \ = \ 0.5$

$Solution$

$y_1(1), y_2(1)$
$x(1) = x(0) + h = 0 + 0.5 = 0.5$
$y_1(1) = y_1(0) + hf_1\big(x(0), y_1(0), y_2(0)\big) = 2 + 0.5 * f_1(0, 2, 4) = 2.5$
$y_2(1) = y_2(0) + hf_2\big(x(0), y_1(0), y_2(0)\big) = 4 + 0.5 * f_2(0, 2, 4) = -4$

$y_1(2), y_2(2)$
$x(2) = x(1) + h = 0.5 + 0.5 = 1$
$y_1(2) = y_1(1) + hf_1\big(x(1), y_1(1), y_2(1)\big) = 2.5 + 0.5 * f_1(0.5, 2.5, -4) = 1.516$
$y_2(2) = y_2(1) + hf_2\big(x(1), y_1(1), y_2(1)\big) = -4 + 0.5 * f_2(0.5, 2.500, -4) = -14$

| i | x | $y_{1(i)}$ | $y_{2(i)}$ |
|---|---|---|---|
| 0 | 0 | 2 | 4 |
| 1 | 0.5 | 2.5 | -4 |
| 2 | 1 | 1.516 | -14 |

4-

$\frac{d^2y}{dx^2} = x\,\frac{dy}{dx} + y^2\,, \qquad y(0) = 1, \frac{dy}{dx}(0) = 2,$

Rearrange equation

$$\frac{d^2y}{dx^2} - x\,\frac{dy}{dx} - y^2 = 0$$

***Solution***

$n = 2$

$a_0(x, y_1) = -y_1^2$

$a_1(x, y_1) = -x$

$a_2(x, y_1) = 1$

$\frac{dy_k}{dx} = y_{k+1} \quad where\ k = 1\ to\ 2-1$

$\frac{dy_1}{dx} = y_2 \quad y_1(0) = 1$

$$\frac{dy_2}{dx} = -\frac{1}{a_2(x, y_1)}\left[\, a_0(x, y_1) + a_1(x, y_1) y_2\right]$$

$$\frac{dy_2}{dx} = -\frac{1}{1}[-y_1^2 - xy_2] = y_1^2 + xy_2, \qquad y_2(0) = 2$$

*Given*

$f_1(x, y_1, y_2) = y_2 - y_1$

$f_2(x, y_1, y_2) = y_1 - y_2$

$x(0) = 0$

$y_1(0) = 1$

$y_2(0) = 2$

$x_f = 0.2$

$h = 0.1$

*Solution*

$y_1(1), y_2(1)$

$x(1) = x(0) + h = 0 + 0.1 = 0.1$

$$x(m) = x(0) + \frac{h}{2} = 0 + \frac{0.1}{2} = 0.05$$

$$y_{1(m)} = y_1(0) + \frac{h}{2} f_1\big(x(0), y_1(0), y_2(0)\big) = 1 + \frac{0.1}{2} f_1(0, 1, 2) = 1.1$$

$$y_{2(m)} = y_2(0) + \frac{h}{2} f_2\big(x(0), y_1(0), y_2(0)\big) = 2 + \frac{0.1}{2} f_2(0, 1, 2) = 2.05$$

$$y_1(1) = y_1(0) + h f_1\big(x(m), y_{1(m)}, y_{2(m)}\big) = 1 + 0.1\, f_1(0.05, 1.100, 2.050) = 1.205$$

$$y_2(1) = y_2(0) + h f_2\big(x(m), y_{1(m)}, y_{2(m)}\big) = 2 + 0.1\, f_2(0.05, 1.100, 2.050) = 2.131$$

$y_1(2), y_2(2)$

$x(2) = x(1) + h = 0.1 + 0.1 = 0.2$

$$x(m) = x(1) + \frac{h}{2} = 0.1 + \frac{0.1}{2} = 0.15$$

$$y_{1(m)} = y_1(1) + \frac{h}{2} f_1\big(x(1), y_1(1), y_2(1)\big) = 1.205 + \frac{0.1}{2} f_1(0.1, 1.205, 2.131) = 1.312$$

$$y_{2(m)} = y_2(1) + \frac{h}{2} f_2\big(x(1), y_1(1), y_2(1)\big) = 2.131 + \frac{0.1}{2} f_2(0.1, 1.205, 2.131) = 2.215$$
$$y_1(2) = y_1(1) + h f_1\big(x(m), y_{1(m)}, y_{2(m)}\big) = 1.205 + 0.1\, f_1(0.15, 1.312, 2.215) = 1.426$$
$$y_2(2) = y_2(1) + h f_2\big(x(m), y_{1(m)}, y_{2(m)}\big) = 2.131 + 0.1\, f_2(0.15, 1.312, 2.215) = 2.336$$

| i | x | y | dy/dx |
|---|---|---|---|
| 0 | 0 | 1 | 2 |
| 1 | 0.1 | 1.205 | 2.131 |
| 2 | 0.2 | 1.426 | 2.336 |